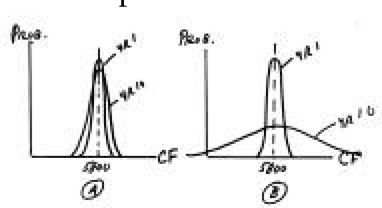
Capital Budgeting

Risk and Uncertainty

Risk and Uncertainty

- Risk ➔ the possibility that actual returns will deviate from expected returns
- Risk ➔ situations in which a probability distribution of possible outcomes can be estimated
- Uncertainty ➔ worse, not enough information available

Probability distribution of expected outcomes



Initial measure of risk

Standard deviation of expected cash flows s

$$s = \sqrt{\sum_{j=1}^{m} (CF_j - \overline{CF})^2 P_j}$$

$$\overline{CF} = \sum_{j=1}^{m} CF_j P_j$$

m = number of possible outcomes

CF_j = j^{th} possible outcome

P_j = probability of CF_j occurring

Improved measure of risk

Coefficient of variation (cv) puts dispersion on a relative basis

$$\text{cv} = \frac{s}{\overline{CF}}$$

Consider $s_x = 300$ and $\overline{CF}_x = 1000$ versus $s_y = 300$ and $\overline{CF}_y = 4000$

Intuitively x is riskier. Need to show that.

$$cv_x = \frac{300}{1000} = .300 \text{ while } cv_y = \frac{300}{4000} = .075$$

Forecasted cash flows

	State of Economy	**CF_j**	**P_j**
j = 1	Recession	100	30%
j = 2	Normal	300	50%
j = 3	Boom	800	20%

Computing coefficient of variation

$$\overline{CF} = .30(100) + .50(300) + .20(800) = 340$$
$$s = \sqrt{(100-340)^2 .30 + (300-340)^2 .50 + (800-340)^2 .20}$$
$$s = 245.76$$
$$CV = \frac{s}{\overline{\overline{CF}}} = \frac{245.76}{340} = .72$$

Required hurdle rate k'

$$k' = f(risk) = f\left(\frac{s}{\overline{\overline{CF}}}\right)$$

Required rate of return k' is a function of the forecasted risk of the project

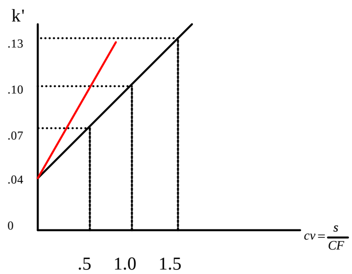
"Penalize" a riskier project by requiring a higher hurdle rate for it to be acceptable

Alternate methods for incorporating risk into capital budgeting

(1) Risk-adjusted discount rate

(2) Certainty equivalents

Risk-adjusted discount rate schedule



Risk-adjusted discount rate

4% is the risk-free rate

Curve is a risk-return trade-off function

Curve is an indifference curve

Firm is indifferent to a cv=.5 and k'=7% or a cv=1.0 and k'=10%

Select k' based on risk from a predetermined schedule and compute NPV

Certainty Equivalents

Convert the expected cash flows to their ***certainty equivalents***

Discount the certainty equivalents at the risk-free rate of interest

Risk-free rate is the yield on a US Treasury bond of the same maturity as the project

Certainty equivalents coefficients

- α_t is the certainty equivalent coefficient for the cash flow at time t
- $0 < \alpha_t = 1$ α_t falls as risk increases
- α_t is determined subjectively by the firm or obtained from a predetermined schedule

$\hat{CF}_t = a_t \overline{CF}_t$ $\hat{CF}_t$ is the certainty equivalent, period t

NPV w/ Certainty Equivalents

Let's say $a_1 = 1.00, a_2 = .95, a_3 = .82, ..., a_{10} = .50$ and $i = 4\%$

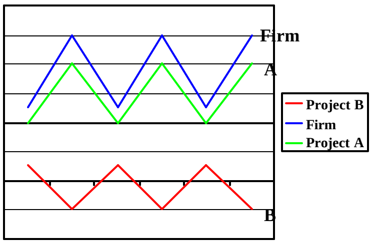
$$NPV = -CF_0 + \frac{a_1(\overline{CF}_1)}{(1.04)^1} + \frac{a_2(\overline{CF}_2)}{(1.04)^2} + \frac{a_3(\overline{CF}_3)}{(1.04)^3} + \cdots + \frac{a_{10}(\overline{CF}_{10})}{(1.04)^{10}}$$

$$NPV = -26000 + \frac{1.00(5800)}{(1.04)^1} + \frac{.95(5800)}{(1.04)^2} + \frac{.82(5800)}{(1.04)^3} + \cdots + \frac{.50(19800)}{(1.04)^{10}}$$

Risk in a portfolio context

- Consider the potential investment, not in isolation (as we have been doing), but in a **portfolio context**
- Look at the relationship between the investment and the firm's existing assets and other potential investments

Two projects and the firm



Which is the more attractive project?

- Project A is cyclical like the overall firm
- Project B is counter cyclical
- In isolation $\sigma_A = \sigma_B$ but ***for this firm***, B is the more attractive project
- Project B is highly negatively correlated with the firm's other assets so addition of Project B reduces overall risk

Correlation and diversification

- Difficult to find projects with high negative correlation
- However, if projects whose returns are uncorrelated are combined, overall risk can be reduced and even eliminated
- Firms seek to diversify into other areas
- Firms try to build a portfolio of assets

Portfolio definitions

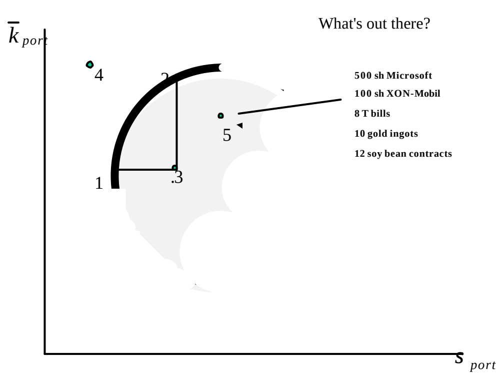
Portfolio ➔ combination of assets

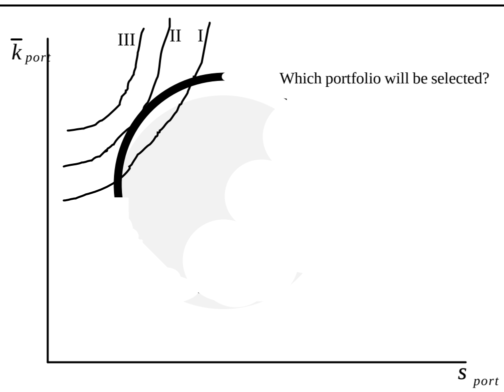
Optimal portfolio ➔ maximum return for a given degree of risk - or - minimum risk for a given rate of return

Opportunity set ➔ all possible portfolios

Efficient frontier ➔ locus of all optimal portfolios

Efficient frontier ➔ dominates all other portfolios of the opportunity set





Risk of an individual asset in a **portfolio context**

$$\text{yield on security j} = \frac{\text{capital gain} + \text{dividend}}{\text{original price}}$$

$$\bar{k}_j = \frac{(P_{j,t+1} - P_{j,t}) + D_{j,t+1}}{P_{j,t}}$$

Risk of an individual asset in a **portfolio context**

Excess return (or risk premium) on security j is the difference between the yield on the security and the yield on risk-free treasury securities (R_f)

$$\text{risk premium on security j} = (\bar{k}_j - R_f) = \frac{(P_{j,t+1} - P_{j,t}) + D_{j,t+1}}{P_{j,t}} - R_f$$

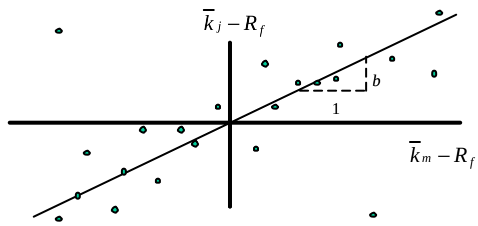
The market's risk premium can be defined similarly:

$$\text{risk premium on the market} = (\bar{k}_m - R_f) = \frac{(P_{m,t+1} - P_{m,t}) + D_{m,t+1}}{P_{m,t}} - R_f$$

Plot the last 60 months of observations

Month	$\bar{k}_j$	$\bar{k}_m$	R_f	$\bar{k}_j - R_f$	$\bar{k}_m - R_f$
1	.05	.06	.03	.02	.03
2	.00	.02	.04	-.04	-.02
3	-.04	-.06	.04	-.08	-.10
4	.09	.08	.04	.05	.04
⋮	⋮	⋮	⋮	⋮	⋮
60	-.01	-.02	.05	-.06	-.07

Characteristic Line



Equation of Characteristic Line

$\bar{k}_j - R_f = a + b[k_m - R_f]$

if $\bar{a} = 0$,

$\bar{k}_j = R_f + b[k_m - R_f]$

Security j's expected return is equal to the risk-free rate plus a risk premium

This risk premium is equal to the market's risk premium times j's beta coefficient

Beta and systematic risk

- Beta is an indicator of **systematic** risk or market risk: interest rate risk, inflation, panics
- $\beta > 1$: stock is aggressive, more volatile than the overall market, e.g., airlines, steel, tires
- $\beta < 1$: stock is defensive, less volatile than the overall market, e.g., utilities

Unsystematic risk

- Variations in security j's return not due to market forces
- Unique to the firm, e.g., financial and operating leverage, managed by crooks
- Eliminated by diversification
- Only systematic risk matters to a firm with a diversified portfolio

What's a firm to do?

McDonald's and Sears are contemplating going into the pizza business

McDonald's is only in fast foods - not diversified;

$\therefore$ they must be concerned with total risk

$k'_{pizza} = f(\frac{s}{\overline{CF}})$ use risk-adjusted discount rate method or certainty equivalents method to find NPV

Sears is diversified into department stores, auto parts, insurance, real estate, etc.;

$\therefore$ they are concerned only with systematic risk

$k'_{pizza} = R_f + b_{pizza}[k_m - R_f]$ use "Beta Model"